3.17-3.20 Review

Name ______________________________ Date ____________________ Period __________

Solve the triangle with the given parts.

1. $\alpha = 28°$, $\beta = 46°$, $c = 17$

$\gamma = 180° - 28° - 46° = \boxed{106°}$

$$\frac{\sin 106°}{17} = \frac{\sin 28°}{a} = \frac{\sin 46°}{b}$$

$$a = \frac{17 \sin 28°}{\sin 106°} = \boxed{8.3}$$

$$b = \frac{17 \sin 46°}{\sin 106°} = \boxed{12.7}$$

Solve the triangle. If there is more than one triangle with the given parts, give both solutions.

2. $\alpha = 41.2°$, $a = 8.1$, $b = 10.6$ $\quad (b > a > h)$ so, 2-triangles

$h = (\sin 41.2°)(10.6) = 6.98$

$\boxed{\text{1st triangle}}$

$$\frac{\sin 41.2°}{8.1} = \frac{\sin \beta_1}{10.6}$$

$$\beta_1 = \sin^{-1}\left(\frac{10.6 \sin 41.2°}{8.1}\right) \approx \boxed{59.5°}$$

$\gamma_1 = 180° - 41.2° - 59.5°$

$\gamma_1 = \boxed{79.3°}$

$$\frac{\sin 41.2°}{8.1} = \frac{\sin 79.3°}{c_1}$$

$$c_1 = \frac{8.1 \sin 79.3°}{\sin 41.2°} \approx \boxed{12.1}$$

2nd triangle

$\beta_2 = 180° - \beta_1 = 180° - 59.5°$
$= \boxed{120.5°}$

$\gamma_2 = 180° - 120.5° - 41.2°$
$= \boxed{18.3°}$

$$\frac{\sin 41.2°}{8.1} = \frac{\sin 18.3°}{c_2}$$

$$c_2 = \frac{8.1 \sin 18.3°}{\sin 41.2°}$$

$c_2 \approx \boxed{3.9}$

3. $\beta = 75.3°$, $b = 12.4$, $c = 9.8$

$b > c$ so, 1 triangle

$$\frac{\sin 75.3°}{12.4} = \frac{\sin \gamma}{9.8}$$

$$\gamma = \sin^{-1}\left(\frac{9.8 \sin 75.3°}{12.4}\right) \approx \boxed{49.9°}$$

$\alpha = 180° - 75.3° - 49.9° = \boxed{54.8°}$

$$\frac{\sin 75.3°}{12.4} = \frac{\sin 54.8°}{a}$$

$$a = \frac{12.4 \sin 54.8°}{\sin 75.3°} \approx \boxed{10.5}$$

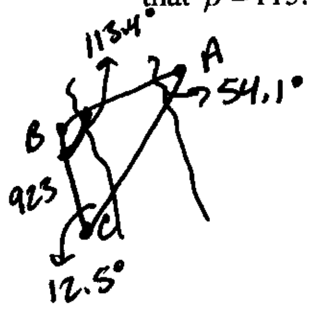
4. $\gamma = 37.9°$, $a = 15.3$, $c = 6.1$

$h = 15.3 \sin 37.9° = 9.39$ $\quad c \not> 9.39$

so, no triangle possible

Solve the problem.

5. To find the distance $\overline{AB}$ across a river, a distance $\overline{BC} = 923$ m is laid off on one side of the river. It is found that $\beta = 113.4°$ and $\gamma = 12.5°$. Find $\overline{AB}$.



$\alpha = 180° - 12.5° - 113.4° = 54.1°$

$$\frac{\sin 54.1°}{923} = \frac{\sin 12.5°}{\overline{AB}}$$

$$\overline{AB} = \frac{923 \sin 12.5°}{\sin 54.1°}$$

$\approx \boxed{246.6\text{ m}}$

Solve the triangle with the given parts.

6. $\gamma = 84.9°$, $a = 7.28$, $b = 8.51$ SAS – Law of Cosines

$c^2 = (7.28)^2 + (8.51)^2 - 2(7.28)(8.51)\cos(84.9°)$

$c^2 = 114.4$

$c \approx \boxed{10.7}$

$\dfrac{\sin 84.9°}{10.7} = \dfrac{\sin\alpha}{7.28}$

$\alpha = \sin^{-1}\left(\dfrac{7.28 \sin 84.9°}{10.7}\right) = \boxed{42.7°}$

$\beta = 180° - 84.9° - 42.7°$

$\beta = \boxed{52.4°}$

7. $a = 6.2$, $b = 12.5$, $c = 13.8$ SSS – Law of Cosines (Find Largest ∠ first!)

$(13.8)^2 = (6.2)^2 + (12.5)^2 - 2(6.2)(12.5)\cos\gamma$

$190.44 = 194.69 - 155\cos\gamma$

$-194.69 \quad -194.69$

$\dfrac{-4.25}{-155} = \dfrac{-155\cos\gamma}{-155}$

$\gamma = \cos^{-1}\left(\dfrac{4.25}{155}\right) \approx \boxed{88.4°}$

$\dfrac{\sin 88.4°}{13.8} = \dfrac{\sin\alpha}{6.2}$

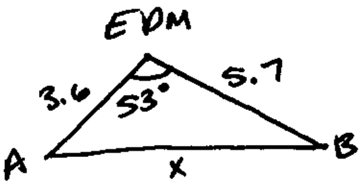
$\alpha = \sin^{-1}\left(\dfrac{6.2 \sin 88.4°}{13.8}\right)$

$\alpha = \boxed{26.7°}$

$\beta = 180° - 88.4° - 26.7° = \boxed{64.9}$

Solve the problem.

8. To find the distance between two small towns, an electronic distance measuring (EDM) instrument is placed on a hill from which both towns are visible. If the distance from the EDM to the towns is 3.6 miles and 5.7 miles and the angle between the two lines of sight is 53°, what is the distance between the towns? Round your answer to the nearest tenth of a mile.



$x^2 = (3.6)^2 + (5.7)^2 - 2(3.6)(5.7)\cos 53°$

$x^2 = 20.75$

$x \approx 4.55 \text{ miles} \approx 4.6 \text{ miles}$

Find the area of triangle ***ABC.***

9. $\alpha = 15.0°$, $b = 10.7$, $c = 7.3$

$A = \frac{1}{2}(10.7)(7.3)\sin(15°) \approx 10.1 \text{ units}^2$

Find the area of the triangle using Heron's formula. Round to the nearest unit.

10. $a = 73.5$, $b = 86.4$, $c = 34.9$

$s = \dfrac{73.5 + 86.4 + 34.9}{2}$

$s = 97.4$

$A = \sqrt{97.4(97.4 - 73.5)(97.4 - 86.4)(97.4 - 34.9)}$

$A = \sqrt{97.4(23.9)(11)(62.5)} \approx \boxed{1265.1 \text{ units}^2}$

Find the magnitudes of the horizontal and vertical components for the vector with the given magnitude and direction angle. Round to an appropriate number of significant digits.

11. $|\mathbf{v}| = 38.6,\ \theta = 77.5°$

$|v_x| = |38.6 \cos 77.5°| = \boxed{8.35}$

$|v_y| = |38.6 \sin 77.5°| = \boxed{37.69}$

Find the component form of the vector with the given magnitude and direction angle.

12. $|\mathbf{v}| = 20.6,\ \theta = 102.5°$

$v_x = 20.6 \cos 102.5° = -4.46$

$v_y = 20.6 \sin 102.5° = 20.11$

→ component form

$\boxed{\langle -4.46, 20.11 \rangle}$

Find the magnitude and direction angle (to the nearest tenth) of the vector. Give the measure of the direction angle as an angle in $[0°, 360°)$.

13. $\langle 8, -8\sqrt{3} \rangle$

magnitude: $\sqrt{(8)^2 + (-8\sqrt{3})^2} = \sqrt{64 + 192} = \sqrt{256} = \boxed{16}$

direction angle: (vector is in ~~4th~~ Quadrant)

$\theta = \tan^{-1}\left(\frac{-8\sqrt{3}}{8}\right) = \tan^{-1}(-\sqrt{3}) = 300°$

Perform the indicated operation. Use the form $\langle a, b \rangle$ for vectors. $\mathbf{u} = \langle -1, 5 \rangle$, $\mathbf{v} = \langle 4, -7 \rangle$

14. Find $3\mathbf{u} - \mathbf{v}$

$3\langle -1, 5 \rangle - \langle 4, -7 \rangle$

$\langle -3, 15 \rangle + \langle -4, 7 \rangle = \boxed{\langle -7, 22 \rangle}$

15. Find $\mathbf{u} \cdot \mathbf{v}$

$\langle -1, 5 \rangle \cdot \langle 4, -7 \rangle$

$(-1)(4) + (5)(-7) = -4 + -35 = \boxed{-39}$

Find the smallest positive angle between the given vectors to the nearest tenth of a degree.

16. $\langle -1, 5 \rangle$ and $\langle 2, 7 \rangle$

$\langle -1, 5 \rangle \to u$, $\langle 2, 7 \rangle \to v$

$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$

$u \cdot v = \langle -1, 5 \rangle \cdot \langle 2, 7 \rangle = -2 + 35 = 33$

$|u| = \sqrt{(-1)^2 + (5)^2} = \sqrt{26}$

$|v| = \sqrt{(2)^2 + (7)^2} = \sqrt{53}$

$\theta = \cos^{-1}\left(\frac{33}{\sqrt{26} \cdot \sqrt{53}}\right) \approx \boxed{27.3°}$

Determine whether the vectors are parallel, perpendicular, or neither.

17. $\langle 2, -4 \rangle$ and $\langle 6, 3 \rangle$

$\frac{-4}{2} \neq \frac{3}{6}$ not parallel

$\langle 2, -4 \rangle \cdot \langle 6, 3 \rangle$

$12 + (-12) = 0 \rightarrow \boxed{\text{perpend.}}$

18. $\langle 9, 1 \rangle$ and $\langle 1, 9 \rangle$

$\frac{1}{9} \neq \frac{9}{1} \rightarrow$ not $\parallel$

$\langle 9, 1 \rangle \cdot \langle 1, 9 \rangle$

$9 + 9 = 18 \rightarrow$ not $\perp$

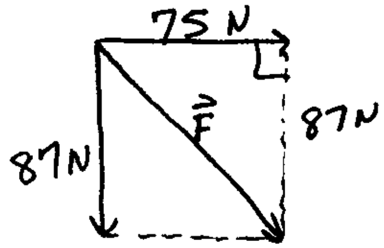
So, neither

19. $\langle -1, 7 \rangle$ and $\langle 3, -21 \rangle$

$\frac{7}{-1} = \frac{-21}{3} \rightarrow$ parallel

Solve the problems.

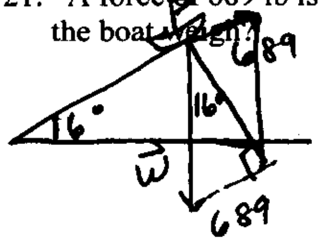
20. One rope pulls a barge due east with a force of 75 N, and another rope pulls the barge due south with a force of 87 N. Find the magnitude of the resultant force acting on the barge.



$$\vec{F}^2 = (75)^2 + (87)^2$$

$$\vec{F} = \sqrt{13{,}194} \approx \boxed{114.9\text{ N}}$$

21. A force of 689 lb is required to pull a boat up a ramp inclined at 16.0° to the horizontal. How much does the boat weigh?

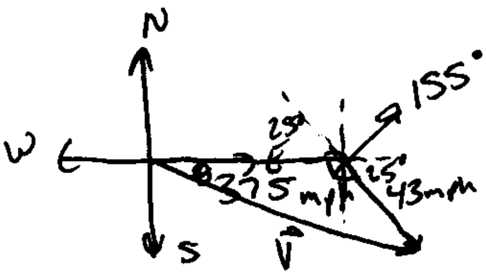


$$\sin 16^\circ = \frac{689}{|\vec{w}|}$$

$$|\vec{w}| = 689 \sin 16^\circ \approx \boxed{2499.7\text{ lb}}$$

22. An airplane flies on a compass heading of 90.0° at 375 mph. The wind affecting the plane is blowing from 295° at 43 mph. What is the true course and ground speed of the airplane? Round results to an appropriate number of significant digits.

SAS: Law of Cosines



$$|\vec{v}^2| = (375)^2 + (43)^2 - 2(375)(43)\cos 155^\circ$$

$$|\vec{v}^2| = 171{,}702.42$$

$$|\vec{v}| = \boxed{414.4\text{ mph}} \rightarrow \text{ground speed!}$$

$$\frac{\sin 155^\circ}{414.4} = \frac{\sin\theta}{43}$$

$$\theta = \sin^{-1}\left(\frac{43\sin 155^\circ}{414.4}\right) \approx 2.5^\circ$$

Course: $90^\circ + 2.5^\circ = \boxed{92.5^\circ}$